



2.3.1 Scalar multiplication

Douglas Wilhelm Harder, LEL, M.Math.

dwharder@uwaterloo.ca

dwharder@gmail.com





Introduction

- In this topic, we will
 - Describe the operation of scalar multiplication
 - First look at real scalar multiplication
 - Next look at complex scalar multiplication
 - Discuss some properties





Real scalar multiplication

- Given a vector in $\mathbf{R}^n$, we can multiply every entry in this vector by a real scalar α
 - This is usually written as $\alpha \mathbf{v}$ but $\mathbf{v} \alpha$ may occasionally be seen

- For example, if $\mathbf{v} = \begin{pmatrix} 1.2 \\ -3.7 \\ 6.4 \\ 0.8 \\ 9.5 \end{pmatrix}$, then $1.5\mathbf{v} = 1.5 \begin{pmatrix} 1.2 \\ -3.7 \\ 6.4 \\ 0.8 \\ 9.5 \end{pmatrix} = \begin{pmatrix} 1.8 \\ -5.55 \\ 9.6 \\ 1.2 \\ 14.25 \end{pmatrix}$





Real scalar multiplication

- We may also just write

$$-4.7 \begin{pmatrix} 8.5 \\ 6.3 \\ -4.9 \end{pmatrix} = \begin{pmatrix} -39.95 \\ -29.61 \\ 23.03 \end{pmatrix}$$

$$\alpha \mathbf{v} = \alpha \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{pmatrix} = \begin{pmatrix} \alpha v_1 \\ \alpha v_2 \\ \alpha v_3 \\ \alpha v_4 \\ \alpha v_5 \\ \alpha v_6 \end{pmatrix}$$





Real scalar multiplication

- Note that,

$$1 \cdot \mathbf{v} = \begin{pmatrix} 1 \cdot v_1 \\ 1 \cdot v_2 \\ 1 \cdot v_3 \\ 1 \cdot v_4 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = \mathbf{v} \quad 0 \cdot \mathbf{v} = \begin{pmatrix} 0 \cdot v_1 \\ 0 \cdot v_2 \\ 0 \cdot v_3 \\ 0 \cdot v_4 \\ 0 \cdot v_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \mathbf{0}_5$$

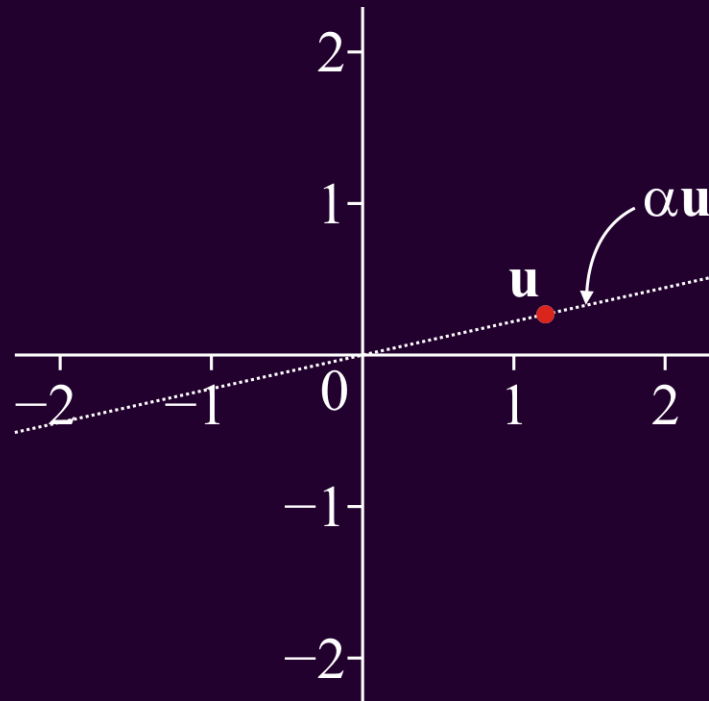
- Also, if $\mathbf{v} = \begin{pmatrix} 1.2 \\ -3.7 \\ 6.4 \\ 0.8 \\ 9.5 \end{pmatrix}$, then $-1 \cdot \mathbf{v} = -1 \cdot \begin{pmatrix} 1.2 \\ -3.7 \\ 6.4 \\ 0.8 \\ 9.5 \end{pmatrix} = \begin{pmatrix} -1.2 \\ 3.7 \\ -6.4 \\ -0.8 \\ -9.5 \end{pmatrix}$





Geometric interpretation

- Scalar multiplication by a real number does not change the *direction* of the vector
 - It stretches, shrinks or reflects the point through the origin





Complex scalar multiplication

- Given a vector in $\mathbf{C}^n$, we can multiply every entry in this vector by a complex scalar α

- For example, if $\mathbf{v} = \begin{pmatrix} 1.2 + 0.3j \\ -3.7 - 1.5j \\ 6.4 + 0.0j \\ 0.8 - 8.2j \\ 9.5 + 1.5j \end{pmatrix}$,
then $(-0.3 + 0.2j)\mathbf{v} = \begin{pmatrix} -0.42 + 0.15j \\ 1.41 - 0.29j \\ -1.92 + 1.28j \\ 1.4 + 2.62j \\ -3.15 + 1.45j \end{pmatrix}$





Complex scalar multiplication

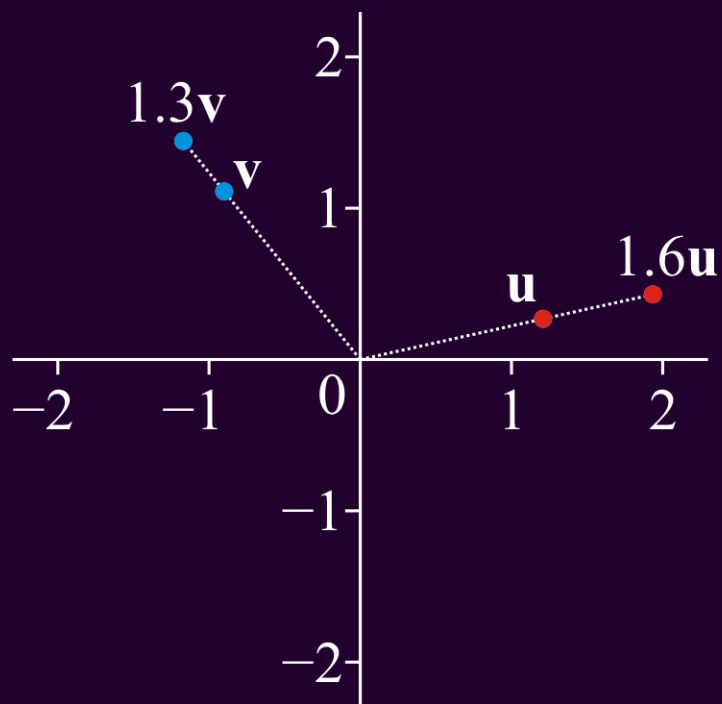
- Note that if $\mathbf{v} \in \mathbf{C}^n$, then it is still true that
$$1\mathbf{v} = \mathbf{v} \text{ and } 0\mathbf{v} = \mathbf{0}_n$$
- Multiplying $\mathbf{v}$ by -1 simply negates the entries, just like with real vectors





Terminology

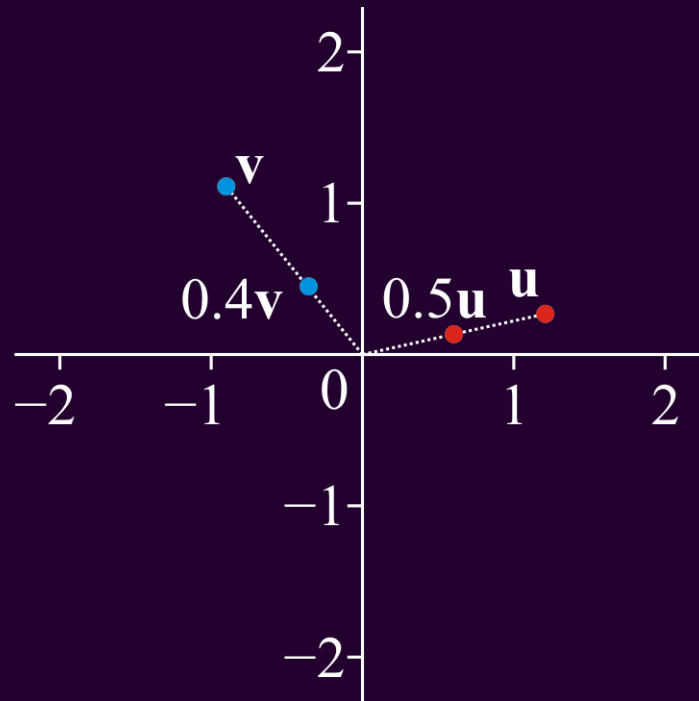
- If $\alpha > 1$, we may occasionally say $\alpha \mathbf{v}$ *amplifies* the vector $\mathbf{v}$





Terminology

- Similarly, if $0 < \alpha < 1$,
we may occasionally say $\alpha \mathbf{v}$ *attenuates* the vector $\mathbf{v}$



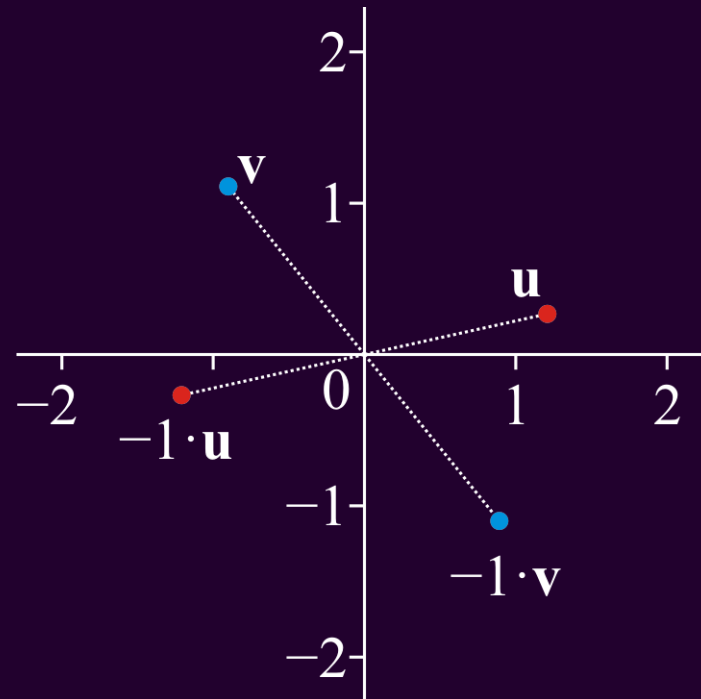
- Engineers tend to want to amplify signals of importance,
while attenuating noise





Terminology

- Occasionally, if $\alpha = -1$,
we may say $-1 \cdot \mathbf{v}$ *mirrors* $\mathbf{v}$ *through the origin*



Do not memorize these terms,
I will reiterate the definitions if I use them.





Observations

- Note that:
 - Multiplying an n -dimensional vector by a scalar results in an n -dimensional vector
 - We are either dealing with real vectors or complex vectors
 - There is no point in asking “What happens if you multiply a real vector by a complex scalar?”
 - It’s like asking “What happens if you divide five cups of water between $1 + j$ people?”





Summary

- Following this topic, you now
 - Understand the concept of scalar multiplication
 - Have seen examples for both real and complex numbers
 - Have been exposed to the terms amplify and attenuate
 - Looked at some properties and made some observations





References

- [1] https://en.wikipedia.org/wiki/Scalar_multiplication





Acknowledgments

None so far.





Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

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for more information.





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